Introduction to Analysis of Variance (ANOVA)

ANOVA is a hypothesis testing procedure that allows us to test the equality of three or more population means in one hypothesis test.

ANOVA is used to test whether the mean response differs between treatment groups in fully randomized and quasi-experimental designs. It is also used to test between-group differences in observational studies when the samples are random.

1. **Formulating the Hypotheses:**

There is only one general form of hypotheses for ANOVA. It is adapted to each particular ANOVA depending on how many groups there are in a particular analysis.

NOTE:

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| **Hypotheses for Analysis of Variance (ANOVA)** |
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| **Answers questions about:** |
| Whether at least one group has a different mean than the others. |
| NOTE:   1. the alternative hypothesis may be stated as   . These two forms of are equivalent to one another. |

1. **The Test Statistic:**

In order to understand the test statistic, we need to know something about the process behind ANOVA. ANOVA estimates the variance of the dependent variable two different ways, and compares those variance estimates using an F test statistic. The two variance estimates are:

1. The between-treatments estimate of the variance, also called the **Mean Square due to Treatments (MSTR)**
   1. The MSTR is the variance estimated under the assumption that is true
2. The within-treatments estimate of the variance, also called the **Mean Square due to Error (MSE)**
   1. The MSE is the variance estimated under the assumption that is false

The test statistic in ANOVA procedures is , and it is the ratio of the two variance estimates: the MSTR and the MSE. The by-hand calculation of this test statistic is the subject of its own handout. The equation is:

with

where

1. **Deciding whether or not to Reject :**

ANOVA tests are always **one-tailed, upper tail tests**. The reason for this is due to the nature of the two variance estimates: the MSTR and the MSE. Here is how ANOVA works. If is actually true and all the means of the groups are equal, then both the MSTR and the MSE will yield similar estimates of the true variance. Therefore, the ratio of MSTR and MSE (which is will be a small number – too small to cause us to reject If, on the other hand, is actually false, and the means of the groups are not all equal, the MSTR will vastly overestimate the true variance, while the MSE will estimate the true variance accurately. Therefore, the ratio of the MSTR and MSE (which is will be a large number – large enough to cause us to reject We only want to reject the null when the difference between MSTR and MSE is large, hence the use of the upper tail test.

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| **When to Reject in ANOVA (the Test Statistic is )** | |
|  | **Always an Upper Tail Test:** |
| **p-value approach:** | Calculate the upper tail of  If the then reject and accept  If the , then do not reject is unsupported. |
| **Critical Value Approach:** | If then reject and accept  If , then do not reject is unsupported. |
| NOTES:   1. is the Test Statistic 2. is the upper tail Critical Value (from the table). 3. The ,   where | |

1. **Interpreting the test:**

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| **How to Interpret an ANOVA:** | |
| **When you:** | **Interpretation:** |
| **Reject** | At the significance level, we can conclude that not all of the population means are equal. *Could also be stated:* At the significance level, we can conclude that at least one of the populations has a different mean than the others. |
| **Do not reject** | At the significance level, we cannot conclude that any of the population means are different from one another. |
| NOTES:   1. When is rejected, you can then perform comparisons of pairs of means from the ANOVA to determine which one is different (or which ones are different) | |

**Assumptions Underlying ANOVA Tests**

ANOVA relies on three assumptions:

1. For each population, the response variable (dependent variable) is normally distributed.
   1. Each treatment group in an experiment represents a potentially different population, and in each population, the response variable, also called the dependent variable, must be normally distributed. This holds true for groups in observational studies as well.
2. The variance of the response variable is the same for all of the populations.
3. The observations must be independent.
   1. In experimental designs, this assumption is satisfied by the use of random assignment to place experimental units into treatment groups.
   2. In observational designs, this assumption is satisfied by using random sampling.